**Research article** 

# PREDICTIVE MODEL TO MONITOR THE DISPERSION OF CHROMIUM AND STREPTOCOCCI INFLUENCED BY POROSITY AND VELOCITY IN SILTY AND FINE SAND FORMATION IN COASTAL AREA OF AMADI-AMA, RIVERS STATE OF NIGERIA.

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# Abstract

Predicting the migration of chromium and streptococci deposition has been evaluated, these parameters are found to predominantly deposit in coastal area of Amadi-Ama, both parameters influences the reaction with deposited pollutants in unconfined bed, these factors defined the rate of influences from environmental challenges that is not thoroughly managed, such conditions were expressed through evaluation of soil and water hazard evaluation, the expressed assessment from formation characteristics discovered homogeneous porosity and velocity to develop higher degree of formation influences, this is to ascertain that the transport of both parameters in coastal area of Amadi-Ama are thoroughly evaluated, the study is essential because it will provide a fundamental base line for experts on the measurement and evaluations including prevention of chromium and streptococci in the study location.

Keywords: predictive model, chromium, streptococci, silty and fine sand

## **1. Introduction**

A major current scientific challenge is scaling from the functional properties of organisms to processes at the ecosystem and global levels (Enquist et al. 2003; Torsvik and Ovreas 2002; Zak et al. 2006 Eluozo and Nwaoburu, 2013). Microbial respiration is a process that has particular importance in the ecosystem and global scales, representing about half of total CO2 flux from soils (Hanson et al. 2000). Furthermore, effects of human-induced climate change on soil microbial communities and their metabolic activities could create potentially devastating feedbacks to the Earth's biosphere (Meir et al. 2006). Biomass made up of fast-growing species respires faster than an equal amount of biomass made up of slow-growing species. Microbes with low growth yields (biomass produced per unit substrate consumed) convert a larger fraction of substrate into CO2 during growth, and so respire faster than efficiently growing organisms. It has been observed that there is an inevitable thermodynamic trade-off between growth rate and yield among heterotrophic organisms (Pfeiffer et al. 2001 Eluozo and Nwaoburu, 2013). Past authors have proposed that two opposing ecological strategies exist at either end of this spectrum: a fast-growing, low yield competitive strategy and a slow growing high yield cooperative strategy (Kreft and Bonhoeffer 2005; Pfeiffer et al. 2001). For microbes, the cooperative, slow, efficient growth strategy is more successful in spatially structured environments such as biofilms (Kreft 2004; Kreft and Bonhoeffer 2005; MacLean and Gudelj 2006; Pfeiffer et al. 2001). With over a billion individual cells and estimates of 104-105 distinct genomes per gram of soil (Gans et al., 2005; Tringe et al., 2005; Fierer et al., 2007b, David et al 2008), bacteria in soil are the reservoirs for much of Earth's genetic biodiversity. This vast phylogenetic and functional diversity can be attributed in part to the dynamic physical and chemical heterogeneity of soil, which results in spatial and temporal separation of microorganisms (Papke and Ward, 2004 Katherineel al 2011). Given the high diversity of carbon (C) - rich compounds in soils, the ability of each taxon to compete for only a subset of resources could also contribute to the high diversity of bacteria in soils through resource partitioning (Zhou et al., 2002). Indeed, Waldrop and Firestone (2004) have demonstrated distinct substrate preferences by broad microbial groups in grassland soils and C resource partitioning has been demonstrated to be a key contributor to patterns of Bacterial co-existence in model communities on plant surfaces (Wilson and Window, 1994 Eluozo and Nwaoburu, 2013).

#### 2. Theoretical background

Niger Delta are influenced by several environmental challenges, there are lots of contamination problem associated from manmade activities including that of natural origins. These situations have generated several soil and water pollution in the study location. Such deltaic challenging demands the need to be addressed, the pollution generation from manmade actions cannot be overemphasized ,such negative impact develop several health challenges on humans. Based on these factors pointed out, it is essential to assess one of the challenging contaminants on humans in the study area. Chromium substance has been discovered to generate high percentage in coastal area of Amadi-Ama. Generation of this pollutant is established through hydrogeological investigations to have deposited in unconfined bed formation. The formation strata, no doubt is a reflected on geological background of the study location

depositing unconfined bed, this can be attributed to formation characteristics, more so in some investigations it develop higher proportion among others influencing chromium and streptococci on migration process in the study area. The integration of this contaminant was established through some hydrological studies previous done, while the formation characteristics were evaluated from standard laboratory experiments using insitu method of sample collection. The investigations produced results but could not generate a stable resolution that can avoid pollution transport from chromium and streptococci, high rate of porosity deposition influence chromium and streptococci in the study location. It has been confirmed from other experts about the deposition influences of microelements, but that of chromium deposit more through manmade activities. The migration of chromium depositing in unconfined bed formation are integrated with streptococci developing some physiochemical reactions, which will be expressed in further migration of the microbes. Focusing these studies, the physiochemical reactions of these parameters is to investigate the rate of concentration on their migration process penetrating unconfined bed. Subject to this relation, the formations setting is influenced by deltaic nature of the strata. There need to generate improved solution that will be used as a baseline in preventing deposition of chromium and streptococci any formation. Mathematical model where found appropriate to apply in other to establish all the state rate of pollution in unconfined beds as observed in the system formulation. This development governing equation will definitely monitor the deposition of those parameters in unconfined beds.

#### **3.** Governing Equation

$$D_L \frac{\partial C}{\partial t} = \overline{V} \frac{\partial^2 C}{\partial Z^2} - \Phi \frac{\partial C}{\partial Z}$$
(1)

Uniformity of deposited porosity in the governing equation and velocity has been expressed from the system, it deposits more influences in unconfined bed, lots of strata variations were investigated, but the stated parameters were found to develop more influence on the movement of chromium and streptococci in the study location, such conditions were considered in the formation of the system that produce the governing equation stated above.

Boundary condition C(o,t) = Co for t > 0 (z,o) and  $(\infty,t) = Co$  for  $t \ge 0$ 

The Laplace transform for a function f(t) which is defined for all values of  $t \ge 0$  is given.

$$\rho f(z) = s\rho = s\rho f(z) - f(o) \text{ where } \rho^{1}(z) = \frac{\partial f}{\partial Z}$$
 (3)

Taking the Laplace transform of the function c with respect to t eqn (1) changes to

$$D_L \rho \left[ \frac{\partial C}{\partial t} \right] = \overline{V} \left[ \frac{\partial^2 C}{\partial Z} \right] - \Phi \rho \frac{\partial C}{\partial Z}$$
(4)

Where  $D_L \rho \left[ \frac{\partial C}{\partial Z} \right] = D_L \rho(c) - C(z, o)$ 

[*C* is a function of *z* and *t* i.e. C(z, t) = f(t), therefore  $\rho f(t) = \rho C(z, f) = \overline{C}$ ]

Let 
$$\overline{C} = D_L \rho(c)$$
 then  $\rho \left[ \frac{\partial C}{\partial Z} \right] = \frac{\partial}{\partial Z} \rho(C) = \frac{\partial \overline{C}}{\partial Z}$  and  $\rho \left[ \frac{\partial^2}{\partial Z^2} \right] = \frac{\partial^2}{\partial Z^2} \rho(c) = \frac{\partial^2 \overline{C}}{\partial Z^2}$ 

Where  $\overline{C}(z) = \rho C(z,t)$ , that is only t changes to s and z is unaffected and s is the Laplace parameter.

At 
$$z = 0: \overline{C}(z) = \int_{0}^{\infty} e^{-st} C(z,t) dt = \int_{0}^{\infty} e^{-st} C_{o} dt = \frac{\infty}{0} \left| -\frac{1}{s} e^{-st} C_{o} \right| = \frac{C_{o}}{s}$$

At 
$$z = \infty$$
:  $\overline{C}(z) = \int_{0}^{\infty} e^{-st} C(z,t) = 0$ 

Therefore at z = 0,  $\overline{C}(z) = \frac{C_o}{s}$ , and at  $z = \infty$ ,  $\overline{C}(z) = 0$ 

[Since this is one dimensional flow equation, partial derivative changes to the full derivative, *s* is a Laplace parameter, which disappears on taking the inverse].

From the substitution Eq

$$D_L s \overline{C} = \overline{\Phi V} \left[ \frac{dc}{\alpha z} \right] - \left[ \frac{d\overline{c}}{dz} \right]$$
(5)

Let  $\overline{C} = Ae^{\lambda z}$  be the solution of the above linear ordinary differential equation. [This is a standard way of solving this class of equations].

The 
$$\frac{d\bar{c}}{dz} = A\lambda e^{\lambda^2}$$
 and  $\frac{d^2\bar{c}}{dz^2} = A\lambda^2 e^{\lambda^2}$  .....(6)

Solution of these values in Eq (5) gives

$$D_L A \lambda^2 e^{\lambda^2} = \overline{\Phi V} A \lambda e^{\lambda^2} - \phi \lambda e^{\lambda^2} \text{ or } \left[ e^{\lambda^2} = \lambda^2 \frac{\overline{\Phi V}}{D_L} \lambda - \frac{s}{D_L} \right] \qquad (7)$$

This will be a solution of the auxiliary equation or the characteristics Equation = 0, this implies that

Equation (8) is the standard quadratic equation and the solution is expressed in this form.

$$\lambda = \frac{\frac{\overline{\Phi V}}{D_L} \pm \sqrt{\frac{\overline{\Phi V}^2}{D_L^2} + \frac{4s}{D_L}}}{2}$$

That is 
$$\lambda_1 = \frac{\overline{\Phi V} + \sqrt{\overline{\Phi V}^2 + 4sD_L}}{2D_L}$$
 and  $\lambda_2 = \frac{\overline{V} - \sqrt{\overline{\Phi V}^2 + 4sD_L}}{2D_L}$ 

Therefore, either  $\overline{C} = Ae^{\lambda, z}$  or  $\overline{C} = Ae^{\lambda^2 z}$  is a solution. However, only the latter satisfies the boundary condition.

At 
$$z = \infty$$
,  $\overline{C} = \frac{C_o}{s}$ ,  $e^{-\infty} = 0$  {because  $\lambda_2$  is -ve and  $\lambda_1$  is +ve}

Therefore  $\overline{C} = A \left[ e^{\frac{\overline{\Phi V} - \sqrt{\overline{\Phi V}^2 + 4sD_L}}{2D_L}} \right]^Z$  is the solution

From Equation (9) C(z,t) can be determined as  $\rho^{-1}\overline{C}(z)$ 

Equation (9) can further be expressed as:

$$C_o \exp\left(\frac{\overline{\Phi V}z}{2D_L}\right) - \frac{1}{\phi s} \exp\left[\frac{-z}{\sqrt{D_L}}\left(\frac{\overline{\Phi V}^2}{4D_L} + s\right)^{\frac{1}{2}}\right]$$

Application of the inverse Laplace transform to the above equation gives

$$C(z,t) = \rho^{-1}\overline{C}(z) = \rho^{-1}\left[C_o \exp\left(\frac{\overline{\Phi V}z}{2D_L}\right) - \frac{1}{s} \exp\left[\frac{-z}{\sqrt{D_L}}\left(\frac{\overline{\Phi V}^2}{4D_L} + s\right)^{\frac{1}{2}}\right]\right]$$
$$= C(z,t) = \rho^{-1}\overline{C}(z) = \rho^{-1}\left[C_o \exp\left(\frac{\overline{\Phi V}z}{2D_L}\right)\rho^{-1}\left[\frac{1}{s} \exp\left[\frac{-z}{\sqrt{D_L}}\left(\frac{\overline{\Phi V}^2}{4D_L} + s\right)^{\frac{1}{2}}\right]\right]\right].....(10)$$

From the Laplace transform table

$$\rho^{-1}\left(\frac{1}{s}\exp\left(-\alpha\sqrt{\beta^2+s}\right)\right) = \int_0^t \frac{\alpha}{2\sqrt{\pi,\beta}}\exp\left[-\left(\frac{\alpha^2}{4u}+\beta^2u\right)du\right] \qquad (11)$$

Here 
$$\frac{Z}{\sqrt{D_L}}$$
 and  $\beta = \frac{\phi \overline{V}}{2\sqrt{D_L}}$ 

Therefore

The term in the bracket = 
$$\left[e^{-\alpha\beta}\int_{0}^{t}\frac{\alpha}{2\sqrt{\pi,\beta}}\exp\left[\frac{(\alpha-2\beta u)^{2}}{4u}du\right]\right]$$
 (13)

$$= e^{-\alpha\beta} \int_{0}^{t} \left[ \frac{\alpha + 2\beta u}{4\sqrt{\pi u^{3}}} + \frac{\alpha - 2\beta u}{4\sqrt{\pi u^{3}}} \right] \exp\left[ -\frac{(\alpha - 2\beta u)^{2}}{4u} du \right]$$
(14)

$$= e^{-\alpha\beta} \left[ \int_{0}^{t} \left[ \frac{\alpha + 2\beta u}{4\sqrt{\pi u^{3}}} \exp\left[ \frac{(\alpha - 2\beta u)^{2}}{4u} du \right] + e^{2\alpha\beta} \int_{0}^{t} \frac{\alpha - 2\beta u}{4\sqrt{\pi u^{3}}} \right] \exp\left[ \frac{(\alpha + 2\beta u)^{2}}{4\sqrt{\pi u^{3}}} du \right] \right] \dots \dots (15)$$

Let 
$$\frac{\alpha - 2\beta u}{\sqrt{4u}} = A$$
 and  $\frac{\alpha + 2\beta u}{\sqrt{4u}} = B$  .....(16)

Differentiating the term in Equation (16) give

$$\frac{dA}{du} = \frac{\sqrt{4u}(0-2\beta) - 2\frac{1}{2}\frac{1}{\sqrt[3]{u}}(\alpha-2\beta u)}{4u} \quad and \quad \frac{\sqrt{4u}(0+2\beta) - 2\frac{1}{2}\frac{1}{\sqrt[3]{u}}(\alpha-2\beta u)}{4u} \quad \dots \dots \quad (17)$$

Or 
$$\frac{dA}{du} = \frac{-4\beta\sqrt{u}\frac{-\alpha}{u} + 2\beta\sqrt{u}}{4u} = \frac{-2\beta u - d}{4\sqrt{u^3}} = \frac{-(\alpha + 2\beta u)}{4\sqrt{u^3}}$$

And 
$$\frac{dB}{du} = \frac{4\beta\sqrt{u}\frac{-\alpha}{u}-2\beta\sqrt{u}}{4u} = \frac{2\beta u-d}{4\sqrt{u^3}} = \frac{-(\alpha-2\beta u)}{4\sqrt{u^3}}$$

Or 
$$dA = \frac{-(\alpha + 2\beta u)}{4\sqrt{u^3}} du$$
 and  $dB = \frac{-(\alpha - 2\beta u)}{4\sqrt{u^3}} du$  .....(18)

For the limit when u = 0

$$A = \frac{\alpha - 2\beta . 0}{0} = \infty B = \frac{\alpha + 2\beta . 0}{0} = \infty, and when$$

$$u = t, A = \frac{\alpha - 2\beta t}{\sqrt{4t}} = and B = \frac{\alpha + 2\beta : t}{\sqrt{4t}} du$$

Changing the integral limits in Equation (19), it is given as

$$\frac{1}{2}\frac{2}{\sqrt{\pi}}e^{-\alpha\beta}\int_{\frac{\alpha-2\beta t}{\sqrt{4t}}}^{\infty}\exp\left(-A^{2}\right)dA + \frac{1}{2}\frac{2}{\sqrt{\pi}}e^{\alpha\beta}\int_{\frac{\alpha-2\beta t}{\sqrt{4t}}}^{\infty}\exp\left(-B^{2}\right)dB \qquad (20)$$

The complimentary error function is defined as  $\operatorname{erfc} x = \frac{2}{\sqrt{\pi}} \int_{\frac{\alpha - 2\beta t}{\sqrt{4t}}}^{\infty} \exp\left(-t^2\right) dt$ 

For which Equation (20) changes to

$$\frac{e^{-\alpha\beta}}{2} \operatorname{erfc} \frac{\alpha - 2\beta t}{\sqrt{4t}} + \frac{e^{-\alpha\beta}}{2} \operatorname{erfc} \frac{\alpha + 2\beta t}{\sqrt{4t}}$$
(21)

The various combinations of  $\alpha$  and  $\beta$  can be simplified as follows:

$$\alpha\beta = \frac{Z}{D_L} \frac{\overline{\Phi V}}{2\sqrt{D_L}} = \frac{\overline{\Phi Vz}}{2\sqrt{D_L}}; \frac{\alpha + 2\beta t}{\sqrt{4t}} = \frac{\overline{\frac{Z}{\sqrt{D_L}}} + \frac{\Phi Vt}{\sqrt{D_L}}}{2\sqrt{t}} = \frac{Z + \overline{\Phi Vt}}{2\sqrt{D_L^{t}}} \text{ And}$$
$$\frac{\alpha - 2\beta t}{\sqrt{4t}} = \frac{\overline{\frac{Z}{\sqrt{D_L}}} + \frac{\overline{\Phi Vt}}{\sqrt{D_L}}}{2\sqrt{t}} = \frac{Z - \overline{\Phi Vt}}{2\sqrt{D_L^{t}}}$$

Using these, equation (21) changes to

$$e\frac{\overline{\Phi V Z}}{\frac{2D_{L}}{2}} \operatorname{erfc}\left[\frac{Z - \overline{\Phi V t}}{2\sqrt{D_{L}^{t}}}\right] + e\frac{\overline{\Phi V Z}}{\frac{2D_{L}}{2}} \operatorname{erfc}\left[\frac{Z + \overline{\Phi V t}}{2\sqrt{D_{L}^{t}}}\right]$$

Therefore finally, Equation (11) with equation (14) changes to

$$C(z,t) = C_o \exp\left(\frac{\overline{\Phi V}z}{2D_L}\right) \frac{1}{2} \exp\left(-\frac{\overline{\Phi V}z}{2D_L}\right) erfc \left[\frac{Z - \overline{\Phi V}t}{2\sqrt{D_L^{t}}}\right] \frac{1}{2} \exp\left(\frac{\overline{\Phi V}z}{2D_L}\right) erfc \left[\frac{Z + \overline{\Phi V}}{2\sqrt{D_L^{t}}}\right]$$
  
Or  $C(z,t) = \frac{C_o}{2} \left[ erfc \left[\frac{Z - \overline{\Phi V}t}{2\sqrt{D_L^{t}}}\right] + \exp\left(\frac{\overline{\Phi V}z}{D_L}\right) erfc \left[\frac{Z + \overline{\Phi V}t}{2\sqrt{D_L^{t}}}\right] \right]$  .....(22)

The expression in [22] generated the final model, numerous mathematical methods have applied in the course of generating the final model, these conditions imply that the developed model has considered numerous phase of the transport system as expressed in the formulation of the system. These conditions produced the governing equations. The developed governing equations ensure that dominant parameters are incorporated so that the developed model can monitor the deposition of chromium and streptococci in the study area.

## 4. Conclusion

Velocity of fluid flow and porosity influences from high degree of void ratio has been assessed to influences the transport of chromium and streptococci in the study location. The developed governing equation were generated from the formulated system, this condition of the variable fashioned out the governing equation, numerous mathematical methods has been applied by other experts, although it has some better solution, but the modeling were only done for a stratum, this developed model are for strata sequential to unconfined bed. The model were generated to ensure that the predominant contaminants are prevented from further migration dispersing the whole area of unconfined bed, the study is essential because different methods that monitor the transport of these contaminants has been developed.

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